VISUALIZATION OF ULTRASONIC FIELDS

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UDC 535.44

The expressions for the intensity distribution of light modulated by an ultrasonic field are analyzed. Phase modulation and modulation in the geometric optics approximation are considered. The effects of both types of modulation on the structure of the optical image are considered on the basis of the Rytov theory of diffraction.

One of the most precise methods of measuring the wavelength and velocity of ultrasound in a medium is that based on the secondary interference of a light wave which has passed though the ultrasonic field [1].

The intensity distribution of the light field in planes beyond a traveling ultrasonic wave when the light is incident normally in the form of a parallel beam is given by the expression [2]:

$$I = \sum_{S=-\infty}^{+\infty} B_S \exp\left[jS\left(\Omega t - \frac{2\pi}{\Lambda} X\right)\right]; \tag{1}$$

$$B_S = \sum_{r=-\infty}^{+\infty} a_r(L)\, a^*_{r-S}(L) \exp\left[j\pi (Z-L) \frac{\lambda}{\Lambda^2} S(S-2r)\right], \tag{2}$$

where λ and Λ are the wavelengths of the light and ultrasound, respectively, in the unperturbed medium, Ω is the ultrasonic frequency, X and Z are the directions of propagation of the ultrasound and light, respectively, L is the depth of the ultrasonic field along the Z axis, t is time, $a_r(L)$ is the amplitude of the light diffracted into the r-th order.

If we apply the elementary Raman–Nath diffraction theory which only takes into account the change in phase of the light beam as it passes through the ultrasonic field [3], we get

$$a_r(L) \sim E_0 J^r(\nu), \tag{3}$$

where E_0 is the amplitude of the incident light field, $J_r(\nu)$ is the r-th-order Bessel function of the first kind, $\nu = 2\pi/\lambda \cdot \Delta n / n_0 L$ is the Raman–Nath parameter which defines the depth of the phase modulation, Δn is the amplitude of the variations in optical refractive index produced by the ultrasound, and n_0 is the optical refractive index of the unperturbed medium.

Substituting (2) and (3) into (1) and using the summation theorem for Bessel functions, we get

$$I = I_0 \sum_{S=-\infty}^{+\infty} J_S[2\nu \sin(\pi b S)] \exp\left[jS\left(\Omega t - \frac{2\pi}{\Lambda} X\right)\right], \tag{4}$$

where

$$I_0 = E_0^2; \quad b = \frac{\lambda}{\Lambda^2}(Z-L).$$

It can be seen from (4) that the periodic structure in the light intensity contains many harmonics at different frequencies. For measuring purposes it is necessary to separate out one of the harmonics of

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 190–195, May–June, 1975. Original article submitted November 4, 1974.

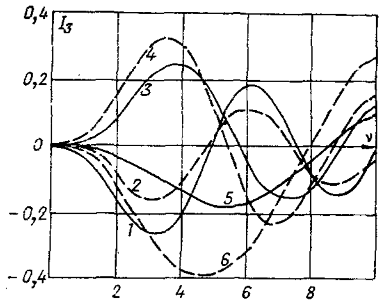

Fig. 1

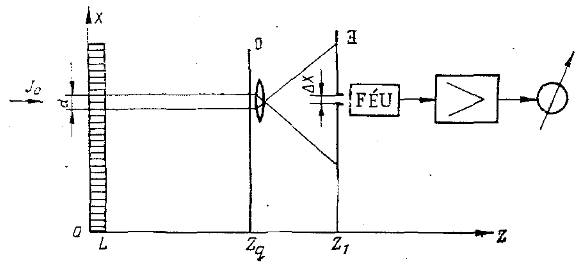

Fig. 2

the ultrasonic frequency. We shall study (4) at the extremum of a parameter in order to determine the planes of best visibility for the third harmonic:

$$\frac{\partial I_3}{\partial b} = 6\pi\nu \cos(3\pi b) \frac{\partial J_3[2\nu \sin(3\pi b)]}{\partial[2\nu \sin(3\pi b)]} = 0;$$

and so

(a) $\cos(3\pi b)=0$, and thus $b={}^1/_6+{}^2/_3\, p$, where $p=0,\ 1;\ 2;\ 3\ldots;$

(b) $$\frac{\partial J_3[2\nu \sin(3\pi b)]}{\partial[2\nu \sin(3\pi b)]} = 0.$$

The first condition is the decisive one in the case of weak ultrasonic fields. It shows that the repetition period and the positions of the planes of best visibility are independent of the modulation depth. The second condition defines the behavior of the optical image in the case of strong ultrasonic waves. For $\nu > 2.2$, the position of the planes of best visibility begins to depend on the phase modulations depth. This fact has been established experimentally by Nomoto [4].

When light travels through an ultrasonic wave, the rays becomes curves at the gradient in refractive index. Thus at the exit from the ultrasonic wave the light field is amplitude-modulated along the X axis. In general, the light intensity can only be calculated if both modulation effects are taken into account. This can be done on the basis of results from general diffraction theory [5]; for the coefficients $a_r(L)$ we have

$$a_r(L) \sim J_r\left[\nu \frac{\sin\frac{u}{2}}{\frac{u}{2}} \exp\left(j\frac{u}{2}\right)\right], \tag{5}$$

where $u = \pi\lambda\Lambda^{-2} L$.

Substituting (5) into (1) and (2), we get

$$I = I_0 \sum_{S=-\infty}^{+\infty} \left(\frac{\operatorname{tg}(\pi b S) - \frac{u}{2}}{\operatorname{tg}(\pi b S) + \frac{u}{2}}\right)^{S/2} J_S\left[2\nu \cos(\pi b S) \sqrt{\operatorname{tg}^2(\pi b S) - \left(\frac{u}{2}\right)^2}\right] \exp\left[jS\left(\Omega t - \frac{2\pi}{\Lambda} X\right)\right]. \tag{6}$$

For $u \to 0$ this equation goes over to (4).

Letting $\lambda \to 0$, we get after some algebra that

$$I = I_0 \sum_{S=-\infty}^{+\infty} J_S\left[S\left(Z - \frac{L}{2}\right) \frac{\Delta n}{n_0} \left(\frac{2\pi}{\Lambda}\right)^2 L\right] \exp\left[jS\left(\Omega t - \frac{2\pi}{\Lambda} X\right)\right]. \tag{7}$$

This expression gives the intensity distribution for small phase modulation and is identical to that obtained in the geometric optics approximation from a calculation of the trajectories of motion of the light rays in an ultrasonic field [6].

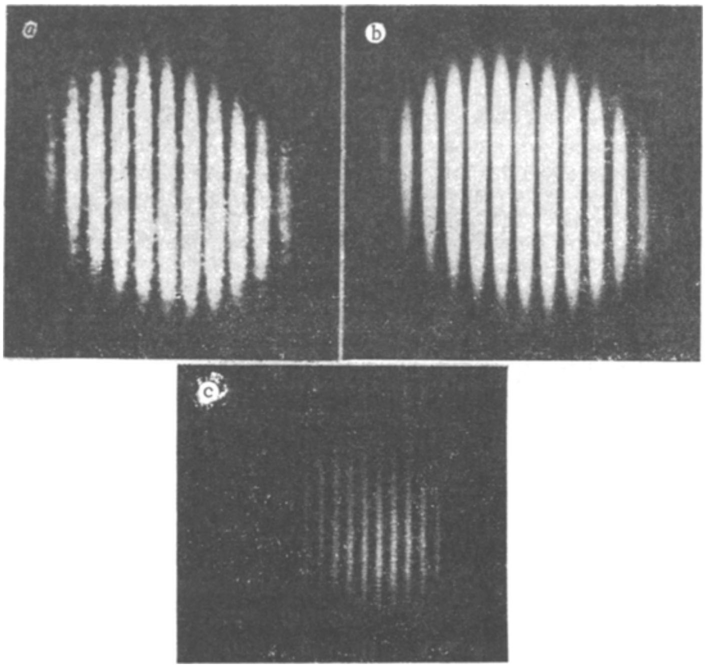

Fig. 3

From a comparison of (4) and (6) we can derive the regions of the parameters Z and ν where it is possible to neglect the curvature of the rays.

We have studied the intensity distribution pattern by calculating the amplitude of the third harmonic of the ultrasonic frequency. We took the following parameter values: $\lambda = 6.28 \cdot 10^{-5}$ cm; $\Lambda = 2.5 \cdot 10^{-2}$ cm; $L = 0.8$ cm; $Z_1 = 11.2$ cm; $Z_2 = 22.3$ cm; $Z_3 = 32.5$ cm; $S = 3$.

The S-th harmonic is formed by the interaction of the r-th and (r + S)-th diffraction waves emerging from the plane Z = L at angles θ_r and θ_{r+S} to the Z axis. We have $\theta_r = r\lambda/\Lambda$.

The condition for a maximum in this harmonic in the plane $Z = Z_\Phi$ is

$$Z_\Phi \cos \theta_r - Z_\Phi \cos \theta_{r+S} = p\lambda,$$

where p is a positive integer.

When $L \lambda\Lambda^{-1} \ll 1$, we get

$$Z_\Phi = \frac{2p\Lambda}{\lambda S\,(2r - S)}. \tag{8}$$

It follows from (8) that the position of the maximum for the S-th harmonic depends to a large extent on the ratio of the amplitudes of the corresponding pair of diffracted waves. In order to get the signal of this harmonic without parasitic phase modulation, we have to choose the value of ν so that a single pair with maximum amplitudes dominates in the diffraction spectrum.

For purely phase modulation it follows from (4) that the phase of the signal in the expansion of the light intensity varies by 180° with a period $Z_\Phi = 2\Lambda^2/\lambda S(2r + S)$ along the Z axis. When there is noticeable curvature of the light rays then the amplitude modulation begins to have a significant affect on the image structure. It can be shown from (7) that for amplitude modulation the phase of the S-th harmonic varies along Z with period $Z_a = \Lambda^2/L \cdot \Delta n/n_0 \cdot {}^1/_4\pi \gg Z_\Phi$. This means that for some values $Z = Z_1$ (Fig. 1) the amplitude of the total signal (curve 1) is greater than that of the signal calculated on the assumption of purely phase modulation (curve 2) over a wide range of values of ν. Thus the phase and amplitude modulation reinforce each other in this case. For other values $Z = Z_2$ (curves 3, 4) and $Z = Z_3$ (curves 5, 6) they tend to cancel each other.

The existence of periodicities in ν for various Z can be explained by the interference of the signals of a given harmonic formed by different orders of the diffraction spectrum.

Reinforcement and cancellation of the phase modulation does not only occur when the light moves beyond the limits of the ultrasonic field, but also inside this field as follows directly from (5). This expression has physical significance when $u < \pi/2$ as far as the ratio of the amplitudes of the light incident on and emerging from the ultrasonic wave is concerned. Nevertheless, it describes the periodicity of the variation in the phase and amplitude modulation as a function of the depth of the ultrasonic field.

The real part of the argument of the Bessel function gives the phase modulation of the light and the imaginary part gives the amplitude modulation. From (5) we can draw the following conclusions:

1. When $u = \pi$ there is no phase modulation. Putting in the value of u, we find that this occurs when $L = \Lambda^2/\lambda$. The imaginary term has its maximum value equal to $4\Delta n\Lambda^2/\lambda^2$.

2. When $u = 2\pi$ there is neither phase nor amplitude modulation. This occurs for $L = 2\Lambda^2/\lambda$. Thus the light emerges from the ultrasonic wave unmodulated in phase and amplitude.

3. It can be shown that the maximum phase modulation occurs when $u = \pi/2$. This corresponds to $L = {}^1/_2\, \Lambda^2/\lambda$ and the real part of (5) is equal to $4\Delta n\Lambda^2/\lambda^2$. The factor for the depth of amplitude modulation is $8/\pi$ times smaller.

If L = 1 cm, for example, the terms which define the depth of the amplitude and phase modulations are equal to 1.57 and $3\Delta n \cdot 10^4$, respectively.

Thus both forms of modulation must be taken into account in calculating the structure of the optical image of the ultrasound.

An important point in the phenomena we have described is that the repetition period of the modulation is independent of the ultrasonic intensity. Similar results were obtained in [7] from a direct solution of the wave equation.

We have made an experimental test of the above relationships by means of photoelectric and photographic recording of the image formed by a light beam passing through an ultrasonic field (Fig. 2). The source of ultrasound was an ITS-19 ceramic radiator of 8-mm diameter with a resonant frequency of 6.0 MHz. The plate was fed from a G3-41 generator and was placed in a vessel of water. The light receiver was an FÉU-51B photomultiplier with a rectangular slit of width 0.1 mm in front of the focusing unit. The optical system was a cylindrical lens which gave a magnification of $\times 10$ in the plane E. The light source was an LG-55 lamp. A camera could be put in place in the plane E. The traveling acoustic waves were photographed by means of stroboscopic lighting obtained from an ML-3 modulator. The experiment was carried out with a voltage of 7 V across the lamp and with $Z_2 = 32.5$ cm. An aperture was placed in the focal plane of the objective O to pass the necessary diffraction orders to the plane E.

Figure 3a shows a system of fringes formed when the entire diffraction spectrum passes through the aperture. The contract of the interference fringes increases when only the first orders are passed (Fig. 3b). When only the second orders are present the structure of the pattern is somewhat different and the fringe spacing is reduced to one half.

The intensity distribution in the fringes (Fig. 3) measured by the photomultiplier corresponds to the shape and width observed visually on the photographs. If the image was formed by phase modulation alone the intensity distribution would have varied harmonically (see Fig. 3b, c). With contrast film the fringes should have a duty ratio of two. This can be observed to some extent in Fig. 3c and also in Fig. 3b for weak ultrasonic waves.

An examination of the fringes (Fig. 3b) shows that the intensity distribution is quite different from harmonic and that the duty ratio is ≈ 4. This indicates that the amplitude modulation is having an important effect. The relative levels of the two different types of modulation can be determined from the nonlinear distortion coefficient of the first-harmonic at the output of the photomultiplier.

When the aperture passes only one diffraction order no fringes should be observed in the plane E according to phase theory. The experiments showed that even for quite low ultrasonic levels signals on several harmonics are observed at the photomultiplier output. This is connected with the effect of ray curvature on the formation of the diffraction spectrum [8].

The ratio of the amplitude of the signal thus formed to that of the signal at the same frequency in the absence of any aperture in the focal plane of the objective can serve as a measure of the effect of the two types of modulation in the formation of the interference fringes. Thus, for example, under the conditions described above the ratio was equal to 0.2 for Z = 100 mm and 0.5 for Z = 300 cm; this is in agreement with the nature of the curves (Fig. 3).

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